Stochastic Block Economic Value Modelling

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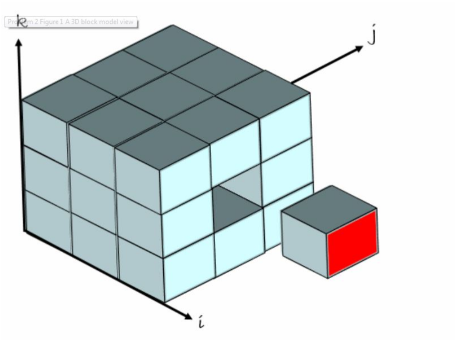
Introduction

Block Economic Value

$$\text{Block Economic Value(BEV)} = \text{Block Revenue} - \text{Cost}$$

$$\text{BEV}_{ij} = [(T_{ij} * G_{ij} * R_{ij} * P_t) - (MC_t + PC_t)]$$



Problem Description

- Develop a model that accounts for the variability in the factors used to calculate BEV

Block Economic Value

$$\text{BEV}_{ij} = [(T_{ij} * G_{ij} * R_{ij} * P_t) - (MC_t + PC_t)] \tag{1}$$

where

- T_{ij} is the tonnage of block B_{ij};
- G_{ij} is the grade of block B_{ij};
- R_{ij} is the recovery of block B_{ij};
- P_t is the price of gold of the block B_{ij};
- MC_t is the mining cost of block B_{ij} at time t;
- PC_t is the cost of processing block B_{ij} at time t.

Fitting Distributions for Parameters

Tonnage (T_{ij}): Does not vary much between the observations:

$$T_{ij} = \bar{t} = \frac{1}{N}\sum_{i=1}^{N} T_{ij} = 624.3$$

Recovery (R_{ij}): Set as a constant as suggested by industry expert

$$R_{ij} = r = 0.9$$

Cost of Mining (C_t): We model costs taking inflation into consideration

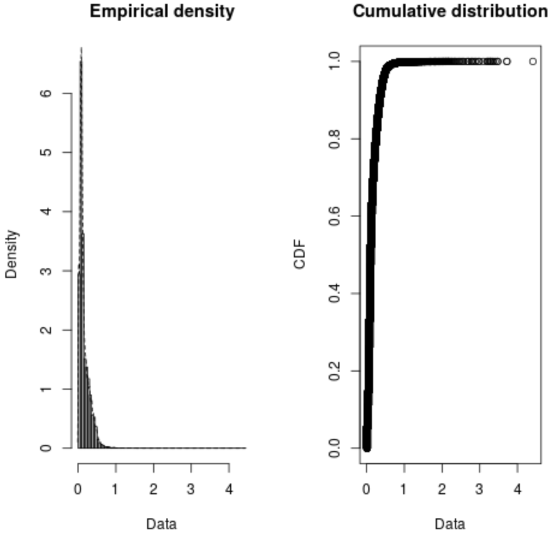
$$\frac{dC}{dt} = \delta C \implies C_t = C_0 e^{\delta t}$$

where

- $C(0) = C_0$ is the initial cost.
- δ is the continuous inflation rate.

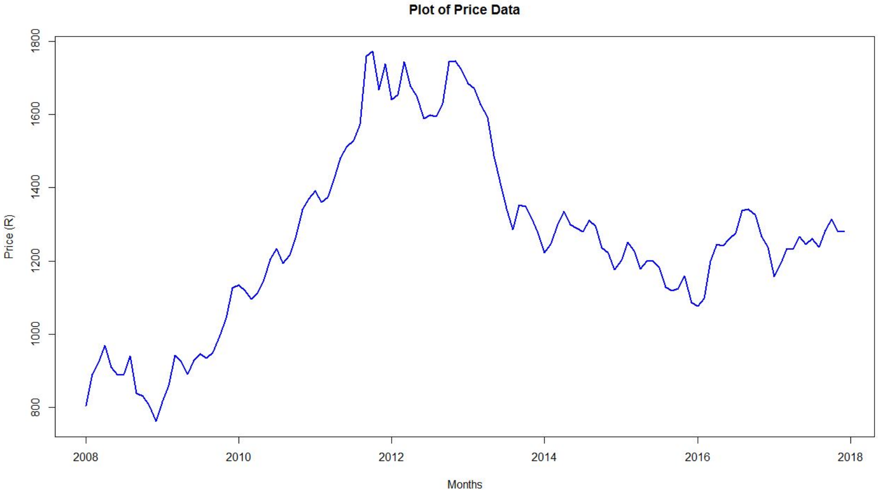
Visual representation of the data

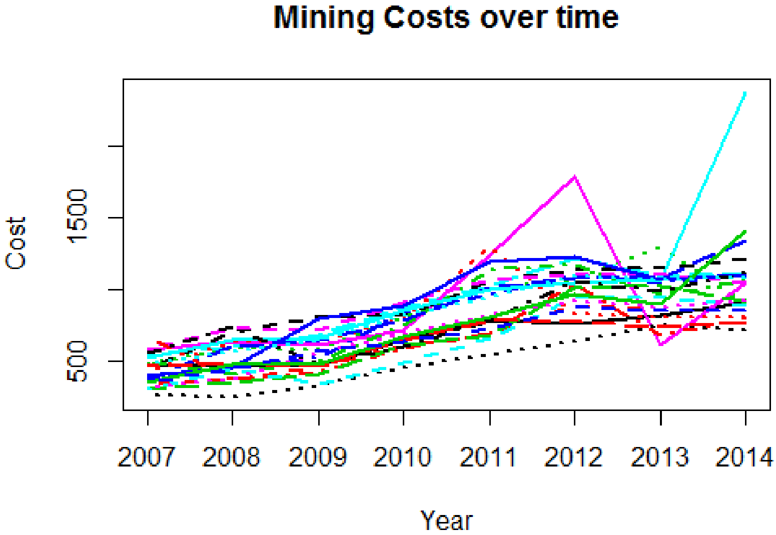
Grade data



Visual representation of the data

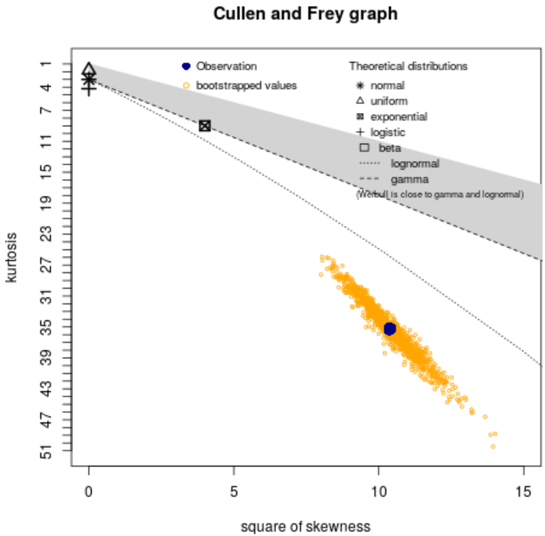
Price change




Mining Costs over time
Cost
500
1500
2007
2008
2009
2010
2011
2012
2013
2014
Year

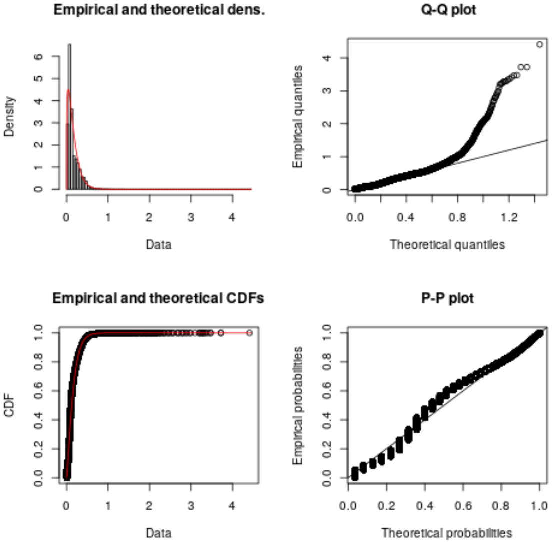
Find possible models for the data

Grade



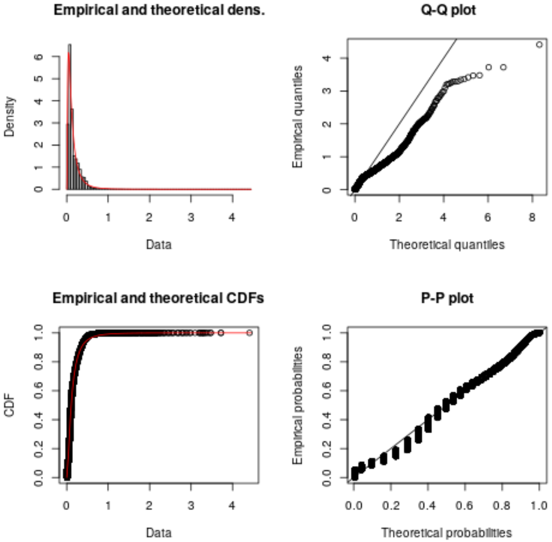
Parametric methods

Weibull distribution



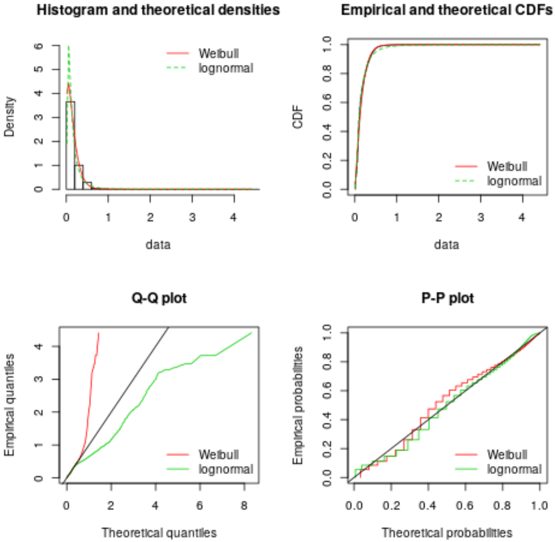
Parametric methods

log-normal distribution



Parametric methods

A comparison



Fitting Distributions for Parameters

- **Grade (G_{ij})**
  We worked with gold data of 302 391 observations.
  We agree with that the lognormal is the best fits the data

$$f(g) = \frac{1}{g\sigma\sqrt{2\pi}} exp\left(-\frac{(\ln g - \mu)^2}{2\sigma^2}\right) \tag{2}$$

where

- $\mu = -2.2366549 \quad (0.001702716)$;
- $\sigma = 0.9363237 \quad (0.001203996)$.

Derivation of Stochastic process for Price of Gold

Define the ratio $\dfrac{\Delta P}{P_t}$ where $\Delta P = P_{t'} - P_t$ and $\Delta t = t' - t$

- Expected annual increase:

$$\mu \Delta t = E\left[\frac{\Delta P}{P_t}\right] \tag{3}$$

  where μ is the increase per unit time.

- Noise:

$$\sigma^2 \Delta t = Var\left(\frac{\Delta P}{P_t}\right) \tag{4}$$

  where σ is the standard deviation.

Derivation of Stochastic process for Price of Gold

- We can therefore construct a simple model.

$$\frac{\Delta P}{P_t} = \mu \Delta t + \gamma(\sigma) \tag{5}$$

  Where $\gamma(\sigma)$ is the noise term.

- To generate noise, we sample from a standard normal distribution.

$$\gamma(\sigma) = \sigma\sqrt{\Delta t}\epsilon, \quad \epsilon \sim N(0,1)$$

We must check that this satisfies the requirements of expectation and variance for our model.

$$E[\gamma] = 0$$
$$Var(\gamma) = \sigma^2 \Delta t$$

Derivation of Stochastic process for Price of Gold

- So the model becomes,

$$\frac{\Delta P}{P_t} = \mu \Delta t + \epsilon \sigma \sqrt{\Delta t}$$

- Writing this as an SDE we have

$$dP_t = P_t[\mu dt + \sigma dB(t)]$$

  Where $B(t)$ is standard Brownian Motion

- We now let $z = log P_t$ and use Ito's Lemma,

$$dF_t = F'(X_t)dX_t + \frac{1}{2}F''(X_t)(dX_t)^2$$

$$dz = \frac{1}{P_t}dP_t + \frac{1}{2}\frac{-1}{P_t^2}(dP_t)^2$$

Derivation of Stochastic process for Price of Gold

- Calculating these terms we have,

$$(dP_t)^2 = \sigma^2 P_t^2 dt$$

  Since $dt^2 = 0$, $dtdB(t) = 0$ and $dB(t)^2 = dt$. These fundamental relationships are known as quadratic variations.

- So we arrive at,

$$dz = \frac{1}{P_t}(P_t[\mu dt + \sigma dB(t)]) - \frac{1}{2P_t^2}\sigma^2 P_t^2 dt$$

$$dz = \left(\mu - \frac{\sigma^2}{2}\right) dt + \sigma dB(t)$$

Integrating gives:

$$lnP_{t'} - lnP_t = \left(\mu - \frac{\sigma^2}{2}\right)(t' - t) + \sigma(B(t') - B(t))$$

$$\implies ln\left(\frac{P_t}{P_0}\right) = \left(\mu - \frac{\sigma^2}{2}\right)t + \sigma\epsilon\sqrt{t}$$

$$\implies P_t = P_0 e^{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma\epsilon\sqrt{t}}$$

Estimating volatility and drift

- **Volatility**

  Using 10 years of data sampled monthly we estimated the volatility and drift of gold.
  Let P the list of all prices and ΔP their associated differences.

$$\begin{aligned}\sigma^2 &= \text{Mean}\left(\frac{\Delta P^2}{P^2 dt}\right)\\ &= 0.0188871\end{aligned}$$

Estimating volatility and drift

- **Drift**

  The log-rate is required in the calculation of the drift.

$$\begin{aligned}
\Delta P e^{\Delta t} &= \frac{P_T}{P_0} \\
&\implies \log \Delta P = \frac{\log(P_T) - \log(P_0)}{\Delta T} \\
&\implies \mu = \frac{\sigma^2}{2} + \log \Delta P \\
&\qquad\quad = 0.13743
\end{aligned}$$

- **Stochastic BEV Model**
  Let b_t be a random variable denoting the block economic value at time t

$$b_t = t * r * g * P_t - C_t \tag{6}$$

  where

  - $t = \frac{1}{N} \sum_{n=1}^{N} t_i$
  - $r = \bar{r}$
  - $g \sim log\mathcal{N}(\mu, \sigma)$
  - $P_t = P_0 \exp\left\{\left(\mu - \frac{1}{2}\sigma^2\right) + \epsilon\sigma\right\}$
  - $\epsilon \sim \mathcal{N}(0, 1)$
  - $C_t = e^{\delta t}$

Conditional distribution

- **Conditional Stochastic BEV Model**
  To calculate the BEV, we define the following conditional distribution:

$$f(b_t|g) = P(b_t < x|\alpha < g < \beta) \tag{7}$$

  Applying Baye's Theorem

$$f(b_t|g) = \frac{P(b_t < x \cap \alpha < g < \beta)}{P(\alpha < g < \beta)} \tag{8}$$

  Taking the expectation yields the Block Economic Value at a particular grade

Implementing the model

- **Conditional Probability**
  Suppose the mining company wishes to determine the probability that a block value will be above x dollars if the grade is between 0.5ppm and 1ppm i.e $Pr(BEV > x|1 > g > 0.5)$.

x Dollars	Probability
1000	0.837739
2000	0.600609
3000	0.430634
4000	0.315498
5000	0.236492
6000	0.180969
7000	0.141001
8000	0.111596
9000	0.0895397
10000	0.0727107

Implementing the model

- **Conditional Expectation**
  This may be useful for various forms of statistical analysis but the company may want a more direct means of evaluating potential prospects.

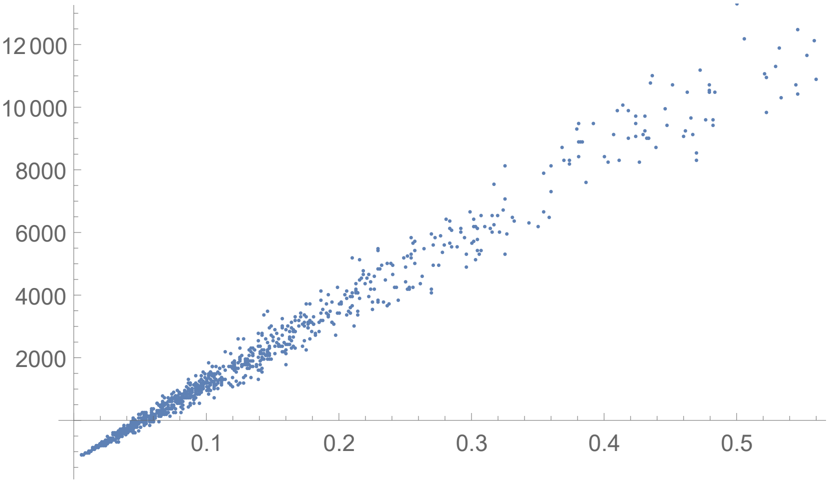
Grade	Expected BEV
(0.1,0.2)	3398.79
(0.2,0.3)	5823.56
(0.3,0.4)	8246.05
(0.4,0.5)	10662.8
(0.5,0.6)	13075.5
(0.6,0.7)	15485.4

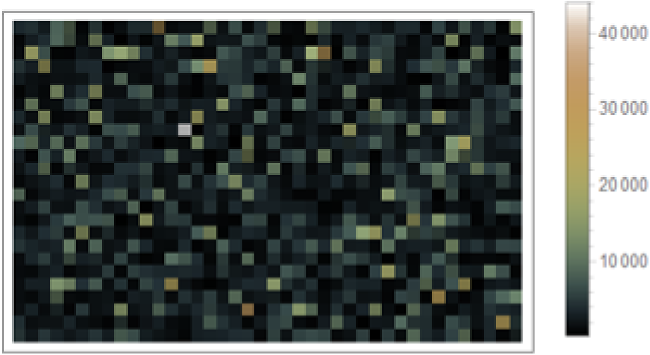
Implementing the model

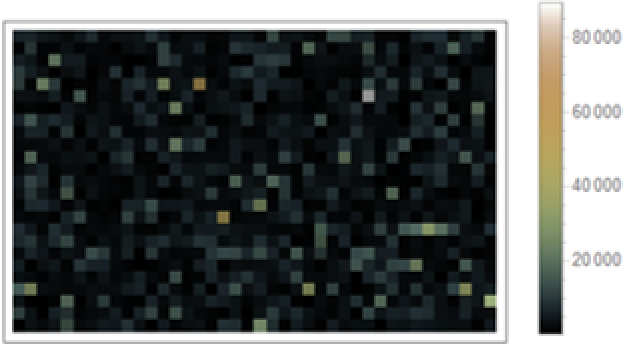
- **Time averages**
  The time average is given by

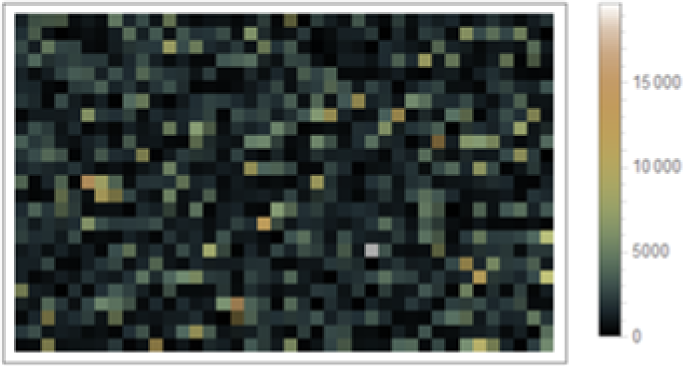
$$\frac{1}{T}\int_0^T f(t)dt$$

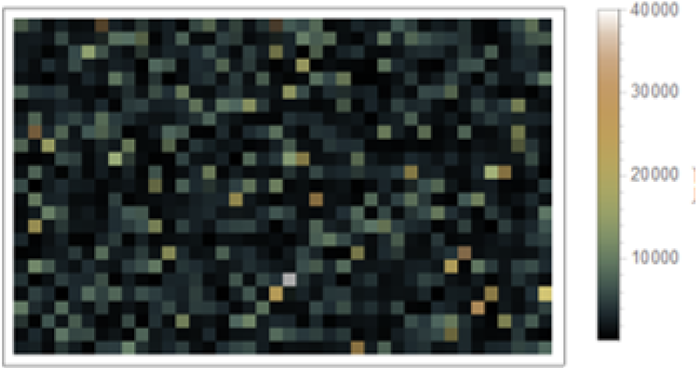
  - Take the time average of $P(t)$.
  - Generate several values of the now time independent P_T.
  - Form a distribution over these values.
  - Take the time average of $C(t)$.
  - Substitute these into the model and generate several values.
  - Plot the BEV against g


12 000
10 000
8000
6000
4000
2000
0.1
0.2
0.3
0.4
0.5


40 000
30 000
20 000
10 000


80 000
60 000
40 000
20 000


15 000
10 000
5000
0


40000
30000
20000
10000

Conclusion

- Our model incorporates two tools parameter fitting and stock modelling.
- Incorporate kriging

References

- Chance, Don M. "The ABCs of Geometric Brownian Motion." Derivatives Quarterly 41 (1994).

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THANK YOU
FOR
YOUR
ATTENTION
ANY QUESTIONS?